0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

10. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

11. A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

12. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

13. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is

(A) $\frac{4}{5}$ (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $\frac{2}{5}$

14. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

(A) $P(A|B) = \frac{P(B)}{P(A)}$ (B) $P(A|B) < P(A)$

(C) $P(A|B) \geq P(A)$ (D) None of these

## 13.6 Random Variables and its Probability Distributions

We have already learnt about random experiments and formation of sample spaces. In most of these experiments, we were not only interested in the particular outcome that occurs but rather in some number associated with that outcomes as shown in following examples/experiments.

(i) In tossing two dice, we may be interested in the sum of the numbers on the two dice.

(ii) In tossing a coin 50 times, we may want the number of heads obtained.

(iii) In the experiment of taking out four articles (one after the other) at random from a lot of 20 articles in which 6 are defective, we want to know the number of defectives in the sample of four and not in the particular sequence of defective and nondefective articles.

In all of the above experiments, we have a rule which assigns to each outcome of the experiment a single real number. This single real number may vary with different outcomes of the experiment. Hence, it is a variable. Also its value depends upon the outcome of a random experiment and, hence, is called random variable. A random variable is usually denoted by X.

If you recall the definition of a function, you will realise that the random variable X is really speaking a function whose domain is the set of outcomes (or sample space) of a random experiment. A random variable can take any real value, therefore, its co-domain is the set of real numbers. Hence, a random variable can be defined as follows :

**Definition 4** A random variable is a real valued function whose domain is the sample space of a random experiment.

For example, let us consider the experiment of tossing a coin two times in succession.

The sample space of the experiment is S = {HH, HT, TH, TT}.

If X denotes the number of heads obtained, then X is a random variable and for each outcome, its value is as given below :

$$X(HH) = 2,\ X(HT) = 1,\ X(TH) = 1,\ X(TT) = 0.$$

More than one random variables can be defined on the same sample space. For example, let Y denote the number of heads minus the number of tails for each outcome of the above sample space S.

Then $$Y(HH) = 2,\ Y(HT) = 0,\ Y(TH) = 0,\ Y(TT) = -2.$$

Thus, X and Y are two different random variables defined on the same sample space S.

**Example 22** A person plays a game of tossing a coin thrice. For each head, he is given Rs 2 by the organiser of the game and for each tail, he has to give Rs 1.50 to the organiser. Let X denote the amount gained or lost by the person. Show that X is a random variable and exhibit it as a function on the sample space of the experiment.

**Solution** X is a number whose values are defined on the outcomes of a random experiment. Therefore, X is a random variable.

Now, sample space of the experiment is

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Then $\quad X(HHH) = \text{Rs } (2 \times 3) = \text{Rs } 6$

$$X(HHT) = X(HTH) = X(THH) = \text{Rs } (2 \times 2 - 1 \times 1.50) = \text{Rs } 2.50$$

$$X(HTT) = X(THT) = (TTH) = \text{Rs } (1 \times 2) - (2 \times 1.50) = -\text{ Re } 1$$

and $\quad X(TTT) = -\text{ Rs } (3 \times 1.50) = -\text{ Rs } 4.50$

where, minus sign shows the loss to the player. Thus, for each element of the sample space, X takes a unique value, hence, X is a function on the sample space whose range is

$$\{-1,\ 2.50,\ -4.50,\ 6\}$$

**Example 23** A bag contains 2 white and 1 red balls. One ball is drawn at random and then put back in the box after noting its colour. The process is repeated again. If X denotes the number of red balls recorded in the two draws, describe X.

**Solution** Let the balls in the bag be denoted by $w_1, w_2, r$. Then the sample space is

$$S = \{w_1\, w_1,\ w_1\, w_2,\ w_2\, w_2,\ w_2\, w_1,\ w_1\, r,\ w_2\, r,\ r\, w_1,\ r\, w_2,\ r\, r\}$$

Now, for $\quad \omega \in S$

$$X(\omega) = \text{number of red balls}$$

Therefore

$$X(\{w_1\, w_1\}) = X(\{w_1\, w_2\}) = X(\{w_2\, w_2\}) = X(\{w_2\, w_1\}) = 0$$

$$X(\{w_1\, r\}) = X(\{w_2\, r\}) = X(\{r\, w_1\}) = X(\{r\, w_2\}) = 1 \text{ and } X(\{r\, r\}) = 2$$

Thus, X is a random variable which can take values 0, 1 or 2.

### 13.6.1 *Probability distribution of a random variable*

Let us look at the experiment of selecting one family out of ten families $f_1, f_2, ..., f_{10}$ in such a manner that each family is equally likely to be selected. Let the families $f_1, f_2, ..., f_{10}$ have 3, 4, 3, 2, 5, 4, 3, 6, 4, 5 members, respectively.

Let us select a family and note down the number of members in the family denoting X. Clearly, X is a random variable defined as below :

$$X(f_1) = 3,\ X(f_2) = 4,\ X(f_3) = 3,\ X(f_4) = 2,\ X(f_5) = 5,$$

$$X(f_6) = 4,\ X(f_7) = 3,\ X(f_8) = 6,\ X(f_9) = 4,\ X(f_{10}) = 5$$

Thus, X can take any value 2,3,4,5 or 6 depending upon which family is selected.

Now, X will take the value 2 when the family $f_4$ is selected. X can take the value 3 when any one of the families $f_1, f_3, f_7$ is selected.

Similarly, $\quad X = 4$, when family $f_2, f_6$ or $f_9$ is selected,

$\quad X = 5$, when family $f_5$ or $f_{10}$ is selected

and $\quad X = 6$, when family $f_8$ is selected.

Since we had assumed that each family is equally likely to be selected, the probability that family $f_4$ is selected is $\frac{1}{10}$.

Thus, the probability that X can take the value 2 is $\frac{1}{10}$. We write $P(X = 2) = \frac{1}{10}$

Also, the probability that any one of the families $f_1, f_3$ or $f_7$ is selected is

$$P(\{f_1, f_3, f_7\}) = \frac{3}{10}$$

Thus, the probability that X can take the value 3 = $\frac{3}{10}$

We write $$P(X = 3) = \frac{3}{10}$$

Similarly, we obtain

$$P(X = 4) = P(\{f_2, f_6, f_9\}) = \frac{3}{10}$$

$$P(X = 5) = P(\{f_5, f_{10}\}) = \frac{2}{10}$$

and $$P(X = 6) = P(\{f_8\}) = \frac{1}{10}$$

Such a description giving the values of the random variable along with the corresponding probabilities is called the *probability distribution of the random variable X*.

In general, the probability distribution of a random variable X is defined as follows:

**Definition 5** The probability distribution of a random variable X is the system of numbers

| | | | | | |
|---|---|---|---|---|---|
| X | : | $x_1$ | $x_2$ | ... | $x_n$ |
| P(X) | : | $p_1$ | $p_2$ | ... | $p_n$ |

where, $$p_i > 0, \quad \sum_{i=1}^{n} p_i = 1, \; i = 1, 2, ..., n$$

The real numbers $x_1, x_2, ..., x_n$ are the possible values of the random variable X and $p_i$ ($i$ = 1,2,..., $n$) is the probability of the random variable X taking the value $x_i$ i.e., $P(X = x_i) = p_i$

**Note** If $x_i$ is one of the possible values of a random variable X, the statement $X = x_i$ is true only at some point (s) of the sample space. Hence, the probability that X takes value $x_i$ is always nonzero, i.e. $P(X = x_i) \neq 0$.

Also for all possible values of the random variable X, all elements of the sample space are covered. Hence, the sum of all the probabilities in a probability distribution must be one.

**Example 24** Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of aces.

**Solution** The number of aces is a random variable. Let it be denoted by X. Clearly, X can take the values 0, 1, or 2.

Now, since the draws are done with replacement, therefore, the two draws form independent experiments.

Therefore,
$$P(X = 0) = P(\text{non-ace and non-ace})$$
$$= P(\text{non-ace}) \times P(\text{non-ace})$$
$$= \frac{48}{52} \times \frac{48}{52} = \frac{144}{169}$$
$$P(X = 1) = P(\text{ace and non-ace or non-ace and ace})$$
$$= P(\text{ace and non-ace}) + P(\text{non-ace and ace})$$
$$= P(\text{ace}).\ P(\text{non-ace}) + P\ (\text{non-ace})\ .\ P(\text{ace})$$
$$= \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52} = \frac{24}{169}$$
and
$$P(X = 2) = P\ (\text{ace and ace})$$
$$= \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

Thus, the required probability distribution is

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | $\frac{144}{169}$ | $\frac{24}{169}$ | $\frac{1}{169}$ |

**Example 25** Find the probability distribution of number of doublets in three throws of a pair of dice.

**Solution** Let X denote the number of doublets. Possible doublets are

(1,1) , (2,2), (3,3), (4,4), (5,5), (6,6)

Clearly, X can take the value 0, 1, 2, or 3.

Probability of getting a doublet $= \frac{6}{36} = \frac{1}{6}$

Probability of not getting a doublet $= 1 - \frac{1}{6} = \frac{5}{6}$

Now $\quad$ P(X = 0) = P (no doublet) = $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$

P(X = 1) = P (one doublet and two non-doublets)

$$= \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$$

$$= 3\left(\frac{1}{6} \times \frac{5^2}{6^2}\right) = \frac{75}{216}$$

P(X = 2) = P (two doublets and one non-doublet)

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} + \frac{1}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{1}{6} \times \frac{1}{6} = 3\left(\frac{1}{6^2} \times \frac{5}{6}\right) = \frac{15}{216}$$

and $\quad$ P(X = 3) = P (three doublets)

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

Thus, the required probability distribution is

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X) | $\frac{125}{216}$ | $\frac{75}{216}$ | $\frac{15}{216}$ | $\frac{1}{216}$ |

**Verification** Sum of the probabilities

$$\sum_{i=1}^{n} p_i = \frac{125}{216} + \frac{75}{216} + \frac{15}{216} + \frac{1}{216}$$

$$= \frac{125 + 75 + 15 + 1}{216} = \frac{216}{216} = 1$$

**Example 26** Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values $x$, has the following form, where $k$ is some unknown constant.

$$P(X = x) = \begin{cases} 0.1, \text{ if } x = 0 \\ kx, \text{ if } x = 1 \text{ or } 2 \\ k(5-x), \text{ if } x = 3 \text{ or } 4 \\ 0, \text{ otherwise} \end{cases}$$

(a) Find the value of $k$.

(b) What is the probability that you study at least two hours ? Exactly two hours? At most two hours?

**Solution** The probability distribution of X is

| X | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| P(X) | 0.1 | $k$ | $2k$ | $2k$ | $k$ |

(a) We know that $\sum_{i=1}^{n} p_i = 1$

Therefore $0.1 + k + 2k + 2k + k = 1$

i.e. $k = 0.15$

(b) P(you study at least two hours) $= P(X \geq 2)$

$= P(X = 2) + P(X = 3) + P(X = 4)$

$= 2k + 2k + k = 5k = 5 \times 0.15 = 0.75$

P(you study exactly two hours) $= P(X = 2)$

$= 2k = 2 \times 0.15 = 0.3$

P(you study at most two hours) $= P(X \leq 2)$

$= P(X = 0) + P(X = 1) + P(X = 2)$

$= 0.1 + k + 2k = 0.1 + 3k = 0.1 + 3 \times 0.15$

$= 0.55$

### 13.6.2 *Mean of a random variable*

In many problems, it is desirable to describe some feature of the random variable by means of a single number that can be computed from its probability distribution. Few such numbers are mean, median and mode. In this section, we shall discuss mean only. Mean is a measure of location or central tendency in the sense that it roughly locates a *middle* or *average value* of the random variable.

**Definition 6** Let X be a random variable whose possible values $x_1, x_2, x_3, ..., x_n$ occur with probabilities $p_1, p_2, p_3, ..., p_n$, respectively. The mean of X, denoted by μ, is the number $\sum_{i=1}^{n} x_i p_i$ i.e. the mean of X is the weighted average of the possible values of X, each value being weighted by its probability with which it occurs.

The mean of a random variable X is also called the expectation of X, denoted by E(X).

Thus, $$E(X) = \mu = \sum_{i=1}^{n} x_i p_i = x_1 p_1 + x_2 p_2 + ... + x_n p_n.$$

In other words, the mean or expectation of a random variable X is the sum of the products of all possible values of X by their respective probabilities.

**Example 27** Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X.

**Solution** The sample space of the experiment consists of 36 elementary events in the form of ordered pairs $(x_i, y_i)$, where $x_i = 1, 2, 3, 4, 5, 6$ and $y_i = 1, 2, 3, 4, 5, 6$.

The random variable X i.e. the sum of the numbers on the two dice takes the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12.

Now $$P(X = 2) = P(\{(1,1)\}) = \frac{1}{36}$$

$$P(X = 3) = P(\{(1,2), (2,1)\}) = \frac{2}{36}$$

$$P(X = 4) = P(\{(1,3), (2,2), (3,1)\}) = \frac{3}{36}$$

$$P(X = 5) = P(\{(1,4), (2,3), (3,2), (4,1)\}) = \frac{4}{36}$$

$$P(X = 6) = P(\{(1,5), (2,4), (3,3), (4,2), (5,1)\}) = \frac{5}{36}$$

$$P(X = 7) = P(\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}) = \frac{6}{36}$$

$$P(X = 8) = P(\{(2,6), (3,5), (4,4), (5,3), (6,2)\}) = \frac{5}{36}$$

$$P(X = 9) = P(\{(3,6), (4,5), (5,4), (6,3)\}) = \frac{4}{36}$$

$$P(X = 10) = P(\{(4,6), (5,5), (6,4)\}) = \frac{3}{36}$$

$$P(X = 11) = P(\{(5,6), (6,5)\}) = \frac{2}{36}$$

$$P(X = 12) = P(\{(6,6)\}) = \frac{1}{36}$$

The probability distribution of X is

| X or $x_i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P(X) or $p_i$ | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |

Therefore,

$$\mu = E(X) = \sum_{i=1}^{n} x_i p_i = 2\times\frac{1}{36} + 3\times\frac{2}{36} + 4\times\frac{3}{36} + 5\times\frac{4}{36}$$

$$+ 6\times\frac{5}{36} + 7\times\frac{6}{36} + 8\times\frac{5}{36} + 9\times\frac{4}{36} + 10\times\frac{3}{36} + 11\times\frac{2}{36} + 12\times\frac{1}{36}$$

$$= \frac{2+6+12+20+30+42+40+36+30+22+12}{36} = 7$$

Thus, the mean of the sum of the numbers that appear on throwing two fair dice is 7.

### 13.6.3 *Variance of a random variable*

The mean of a random variable does not give us information about the variability in the values of the random variable. In fact, if the variance is small, then the values of the random variable are close to the mean. Also random variables with different probability distributions can have equal means, as shown in the following distributions of X and Y.

| X | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| P(X) | $\frac{1}{8}$ | $\frac{2}{8}$ | $\frac{3}{8}$ | $\frac{2}{8}$ |

| Y | –1 | 0 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| P(Y) | $\frac{1}{8}$ | $\frac{2}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |

Clearly $\quad E(X) = 1\times\frac{1}{8}+2\times\frac{2}{8}+3\times\frac{3}{8}+4\times\frac{2}{8}=\frac{22}{8}=2.75$

and $\quad E(Y) = -1\times\frac{1}{8}+0\times\frac{2}{8}+4\times\frac{3}{8}+5\times\frac{1}{8}= 6\times\frac{1}{8}=\frac{22}{8}=2.75$

The variables X and Y are different, however their means are same. It is also easily observable from the diagramatic representation of these distributions (Fig 13.5).

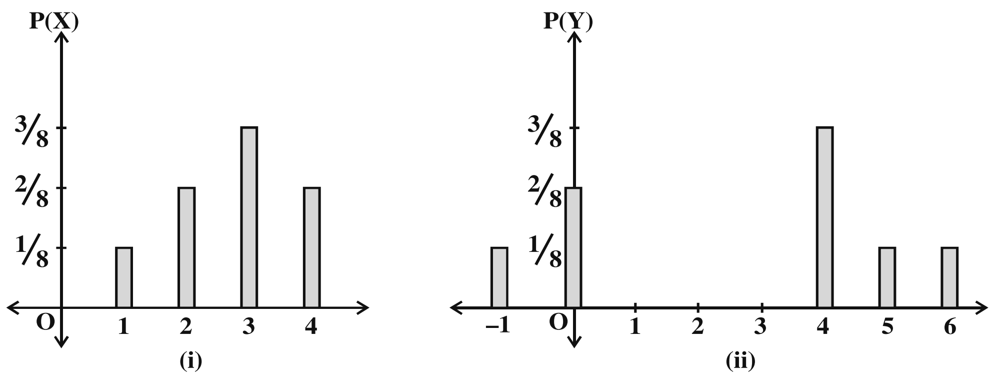


**Fig 13.5**

To distinguish X from Y, we require a measure of the extent to which the values of the random variables spread out. In Statistics, we have studied that the variance is a measure of the spread or scatter in data. Likewise, the variability or spread in the values of a random variable may be measured by variance.

**Definition 7** Let X be a random variable whose possible values $x_1, x_2,...,x_n$ occur with probabilities $p(x_1), p(x_2),..., p(x_n)$ respectively.

Let $\mu = E(X)$ be the mean of X. The variance of X, denoted by Var (X) or $\sigma_x^2$ is defined as

$$\sigma_x^2 = \text{Var}(X) = \sum_{i=1}^{n}(x_i-\mu)^2\, p(x_i)$$

or equivalently $\quad \sigma_x^2 = E(X-\mu)^2$

The non-negative number

$$\sigma_x = \sqrt{\text{Var(X)}} = \sqrt{\sum_{i=1}^{n}(x_i - \mu)^2\, p(x_i)}$$

is called the *standard deviation* of the random variable X.

**Another formula to find the variance of a random variable.** We know that,

$$\text{Var (X)} = \sum_{i=1}^{n}(x_i - \mu)^2\, p(x_i)$$

$$= \sum_{i=1}^{n}(x_i^2 + \mu^2 - 2\mu\, x_i)\, p(x_i)$$

$$= \sum_{i=1}^{n} x_i^2\, p(x_i) + \sum_{i=1}^{n}\mu^2\, p(x_i) - \sum_{i=1}^{n} 2\mu x_i\, p(x_i)$$

$$= \sum_{i=1}^{n} x_i^2\, p(x_i) + \mu^2 \sum_{i=1}^{n} p(x_i) - 2\mu \sum_{i=1}^{n} x_i\, p(x_i)$$

$$= \sum_{i=1}^{n} x_i^2\, p(x_i) + \mu^2 - 2\mu^2 \left[\text{since} \sum_{i=1}^{n} p\,(x_i) = 1 \text{ and } \mu = \sum_{i=1}^{n} x_i\, p(x_i)\right]$$

$$= \sum_{i=1}^{n} x_i^2\, p(x_i) - \mu^2$$

or $$\text{Var (X)} = \sum_{i=1}^{n} x_i^2\, p(x_i) - \left(\sum_{i=1}^{n} x_i\, p(x_i)\right)^2$$

or $$\text{Var (X)} = E(X^2) - [E(X)]^2, \text{ where } E(X^2) = \sum_{i=1}^{n} x_i^2\, p(x_i)$$

**Example 28** Find the variance of the number obtained on a throw of an unbiased die.

**Solution** The sample space of the experiment is S = {1, 2, 3, 4, 5, 6}.

Let X denote the number obtained on the throw. Then X is a random variable which can take values 1, 2, 3, 4, 5, or 6.

Also $$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$$

Therefore, the Probability distribution of X is

| X | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| P(X) | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

Now $$E(X) = \sum_{i=1}^{n} x_i\, p(x_i)$$

$$= 1\times\frac{1}{6}+2\times\frac{1}{6}+3\times\frac{1}{6}+4\times\frac{1}{6}+5\times\frac{1}{6}+6\times\frac{1}{6}=\frac{21}{6}$$

Also $$E(X^2) = 1^2\times\frac{1}{6}+2^2\times\frac{1}{6}+3^2\times\frac{1}{6}+4^2\times\frac{1}{6}+5^2\times\frac{1}{6}+6^2\times\frac{1}{6}=\frac{91}{6}$$

Thus, $$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$= \frac{91}{6}-\left(\frac{21}{6}\right)^2 = \frac{91}{6}-\frac{441}{36} = \frac{35}{12}$$

**Example 29** Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.

**Solution** Let X denote the number of kings in a draw of two cards. X is a random variable which can assume the values 0, 1 or 2.

Now $$P(X = 0) = P(\text{no king}) = \frac{{}^{48}C_2}{{}^{52}C_2} = \frac{\dfrac{48!}{2!(48-2)!}}{\dfrac{52!}{2!(52-2)!}} = \frac{48\times 47}{52\times 51} = \frac{188}{221}$$

$$P(X = 1) = P(\text{one king and one non-king}) = \frac{{}^{4}C_1\ {}^{48}C_1}{{}^{52}C_2}$$

$$= \frac{4\times 48\times 2}{52\times 51} = \frac{32}{221}$$

and $\qquad P(X = 2) = P\text{ (two kings)} = \dfrac{^4C_2}{^{52}C_2} = \dfrac{4\times 3}{52\times 51} = \dfrac{1}{221}$

Thus, the probability distribution of X is

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | $\frac{188}{221}$ | $\frac{32}{221}$ | $\frac{1}{221}$ |

Now $\qquad$ Mean of $\quad X = E(X) = \sum_{i=1}^{n} x_i\, p(x_i)$

$$= 0\times\frac{188}{221} + 1\times\frac{32}{221} + 2\times\frac{1}{221} = \frac{34}{221}$$

Also $\qquad E(X^2) = \sum_{i=1}^{n} x_i^2\, p(x_i)$

$$= 0^2\times\frac{188}{221} + 1^2\times\frac{32}{221} + 2^2\times\frac{1}{221} = \frac{36}{221}$$

Now $\qquad \text{Var}(X) = E(X^2) - [E(X)]^2$

$$= \frac{36}{221} - \left(\frac{34}{221}\right)^2 = \frac{6800}{(221)^2}$$

Therefore $\qquad \sigma_x = \sqrt{\text{Var}(X)} = \dfrac{\sqrt{6800}}{221} = 0.37$

## EXERCISE 13.4

1. State which of the following are not the probability distributions of a random variable. Give reasons for your answer.

(i)

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | 0.4 | 0.4 | 0.2 |

(ii)

| X | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| P(X) | 0.1 | 0.5 | 0.2 | – 0.1 | 0.3 |

(iii)

| Y | – 1 | 0 | 1 |
|---|---|---|---|
| P(Y) | 0.6 | 0.1 | 0.2 |

(iv)

| Z | 3 | 2 | 1 | 0 | –1 |
|---|---|---|---|---|---|
| P(Z) | 0.3 | 0.2 | 0.4 | 0.1 | 0.05 |

**2.** An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X? Is X a random variable ?

**3.** Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

**4.** Find the probability distribution of

(i) number of heads in two tosses of a coin.

(ii) number of tails in the simultaneous tosses of three coins.

(iii) number of heads in four tosses of a coin.

**5.** Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

(i) number greater than 4

(ii) six appears on at least one die

**6.** From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

**7.** A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

**8.** A random variable X has the following probability distribution:

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| P(X) | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2+k$ |

Determine

(i) $k$ (ii) P(X < 3)

(iii) P(X > 6) (iv) P(0 < X < 3)

**9.** The random variable X has a probability distribution P(X) of the following form, where $k$ is some number :

$$P(X) = \begin{cases} k, & if \quad x = 0 \\ 2k, & if \quad x = 1 \\ 3k, & if \quad x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) Determine the value of $k$.

(b) Find P (X < 2), P (X ≤ 2), P(X ≥ 2).

**10.** Find the mean number of heads in three tosses of a fair coin.

**11.** Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

**12.** Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X).

**13.** Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

**14.** A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X.

**15.** In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X) and Var (X).

Choose the correct answer in each of the following:

**16.** The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

(A) 1 (B) 2 (C) 5 (D) $\frac{8}{3}$

**17.** Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is

(A) $\frac{37}{221}$ (B) $\frac{5}{13}$ (C) $\frac{1}{13}$ (D) $\frac{2}{13}$